Übung Formale Grundlagen

Sets

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Definition (Set (naive))

A set is a collection of distinct, unordered, potentially unbounded, abstract objects

Example

- $\{2, 5, 17\}$
- {red, blue, black, yellow}
- {x, y, z, w}
- {Leipzig, Berlin, Hamburg, Magdeburg}
- {142, α, 🚲, {rose, orchid}}
- $\{x \mid x$ is the president of France$\}$

Notation & Terminology

- **Member:** $x \in X$, 'x is a member/element of X'
- **Non-member:** $x \notin X$, 'x is not a member of X'
- **Singleton:** A set containing one element {Tolkien}
- **Null set, empty set:** Contains no elements $\emptyset, \{\}$
- **Recursion:** Applying a procedure includes applying the procedure itself
- **List notation:** Curly brackets "{}", list of members $X = \{1, 2, 3\}$
- **Predicate notation:** Indicate a property the members share $X = \{x | x > 2\}$
- **Ordering:** Sets are *unordered*, $\{1, 2, 3\}$ is the same as $\{3, 2, 1\}$

Definition (Set Equality)

Let A and B be sets. The set A is equal to the set B if all members of A are members of B and all members of B are members of A. We write $A = B$.

Definition (Inclusion)

Let A and B be sets. We say A is a **subset** of B (denoted by $A \subseteq B$) if every member of the set A is a member of the set B.
We say B is a **superset** of A (denoted by $B \supseteq A$).
If the set C is not a subset of the set D, we write $C \nsubseteq D$.

Proper Subset

Definition (Proper Subset)

Let A and B be sets. We say A is a **proper subset** of B if A is a subset of B ($A \subseteq B$) and $A \neq B$. We write $A \subset B$.

Exercise 1

Let us consider the following sets:

$$A = \{a, b, c, 2, 3, 4\}$$
$$B = \{a, b\}$$
$$C = \{c, 2\}$$
$$D = \{b, c\}$$
$$E = \{a, b, \{c\}\}$$
$$F = \emptyset$$
$$G = \{\{a, b\}, \{c, 2\}\}$$

Decide if the following statements are true:

1. $c \in A$
2. $c \in F$
3. $c \in E$
4. $\{c\} \in E$
5. $\{c\} \in C$
6. $B \subseteq A$
7. $D \subset A$
8. $A \subseteq C$
9. $B \in G$
10. $B \subseteq G$
11. $B \subset G$
12. $\{B\} \subseteq G$
13. $G \subseteq A$
14. $D \subseteq A$
15. $\{\{c\}\} \subseteq E$