Übung Formale Grundlagen

Sets 2

Ondřej Smetana

ondrej.smetana@uni-leipzig.de

29. 10. 2026
04-006-1001.ÜB02

Sets

Notation and Terminology

Definition (Membership)

If the element a is a member of the set (Menge) X, we write $a \in X$.

Definition (Inclusion)

Let A and B be sets. We say A is a **subset** (Untermenge/Teilmenge) of B (denoted by $A \subseteq B$) if every member of the set A is a member of the set B.
We say B is a **superset** (Obermenge) of A (denoted by $B \supseteq A$).

Definition (Proper subset)

Let A and B be sets. We say A is **proper subset** (echte Untermenge) of B ($A \subset B$) if $A \subseteq B$ and $A \neq B$.
"There is an element in B which is not in A."

Subset & Proper Subset

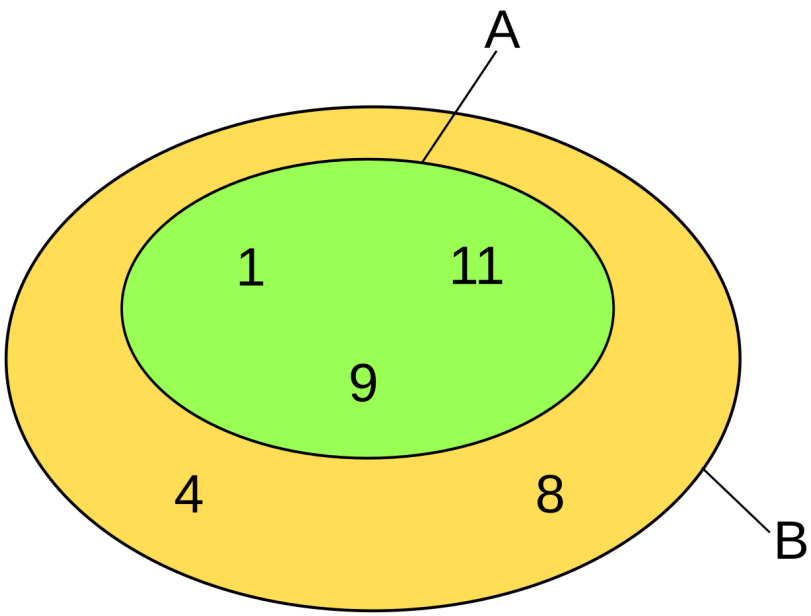


Figure 1: Proper Subset, $A \subset B$

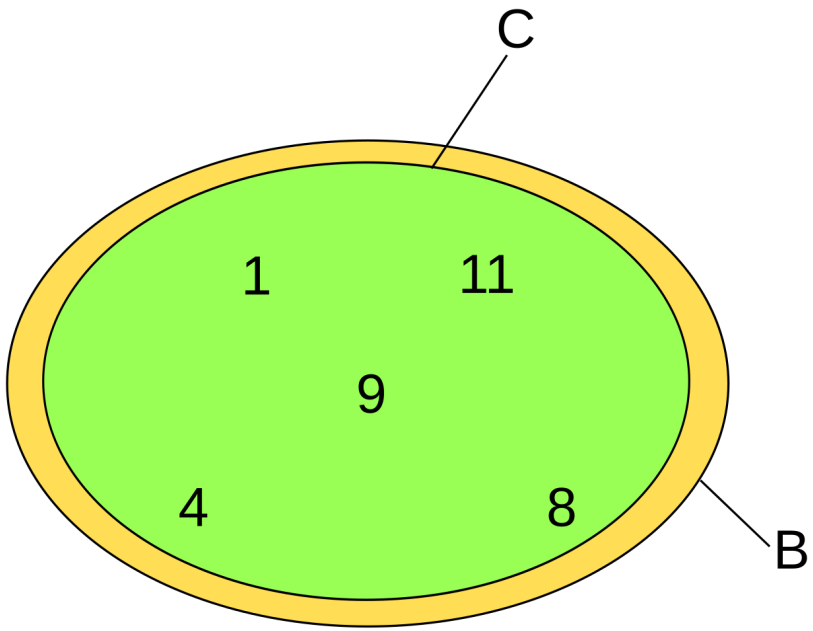


Figure 2: Not a Proper Subset, $C \subseteq B$

Exercise 1

Choose the right symbol(s) ($=, \subseteq, \subset, \supset, \supseteq, \in, \notin, \not\subset, \not\supset$) which represents the relation between two sets (or a set and an element). Think about what symbols cannot be used.

1.	$\{x, y, z\}$		$\{z, z, z, y, x, x\}$
2.	{📄, 🦴, 🎂}		{📄, 🦴}
3.	$\{a, b, \{c, d\}\}$		$\{a, b, \{c, d\}, e, f\}$
4.	$\{\alpha, \beta\}$		$\{\alpha, \beta, \{\alpha, \beta\}\}$
5.	$\emptyset$		$\{1, 2, 3\}$
6.	$\{x\}$		$\{\{x\}\}$

Definition (Cardinality)

Let A be a (finite) set. The **cardinality** (Mächtigkeit/Kardinalität) of A is the number of elements A contains, we write $|A|$.

Exercise 2

What is the cardinality of the set $\{x|x \text{ is a German state}\}$?

Exercise 3

What is the cardinality of the set $\{a, b, \{c, d\}\}$?

Definition (Power Set)

Let X be a set. The **power set** (Potenzmenge) of X is the set of all subsets of X, denoted by $\wp(X)$.

Exercise 4

Write down the power set of {Fry, Leela, Bender}.

Set Operations

Set Union

Definition (Set Union)

Let A and B be sets. The **union** (Vereinigung) of A and B is the set which contains elements from A and from B,

$$A \cup B = \{x | x \in A \text{ or } x \in B\}.$$

Exercise 5

Consider the following sets:

- $L = \{a | a \text{ is a landlocked country}\}$,
- $D = \{a | a \text{ is a democracy}\}$.

Decide if

1. North Korea $\in L \cup D$,
2. Italy $\in L \cup D$,
3. Switzerland $\in L \cup D$,
4. Afghanistan $\in L \cup D$.

Set Intersection

Definition (Set Intersection)

Let A and B be sets. The **intersection** (Schnitt(menge)) of A and B is the set which contains elements found in both A and B,

$$A \cap B = \{x | x \in A \text{ and } x \in B\}.$$

Exercise 6

Consider the following sets:

- $E = \{x | x \text{ is a member of EU}\}$,
- $N = \{x | x \text{ is a member of NATO}\}$,
- $M = \{x | x \text{ has Mediterranean coast}\}$.

Decide if

1. UK $\in E \cap N$,
2. France $\in (E \cap N) \cap M$,
3. Norway $\in E \cap M$,
4. Spain $\in N \cap M$,
5. Austria $\in E \cap N$.

List members of $(E \cap N) \cap M$.

Set Difference

Definition (Set Difference)

Let A and B be sets. The **difference** (Differenz) of A and B is the set which contains members of A which are not members of B,

$$A - B = A \setminus B = \{x | x \in A \text{ and } x \notin B\}.$$

Set Difference

Exercise 7

Consider the following sets of European countries.

- $G = \{x | \text{the official language of } x \text{ is Germanic}\}$,
- $E = \{x | x \text{ uses Euro as currency}\}$,
- $I = \{x | x \text{ is an island nation}\}$.

Write down

1. $G - E$,
2. $E - G$,
3. $G - I$,
4. $I - E$.

Set Complement

Definition (Set Complement)

Let A be a set. The **complement** (Komplement) of A the set that contains everything that is not in A,

$$A' = \{x | x \notin A\}.$$

Careful

When we talk about complements, we *should* specify with respect to what, i.e., what is *the universe of discourse*.

Exercise 8

Consider the sets

- $U = \{$Germany, France, Spain, United Kingdom, Sweden, Switzerland, Iceland, Italy$\}$,
- $G = \{x \in U | $the official language of x is Germanic$\}$,
- $E = \{x \in U | x$ uses Euro as currency$\}$.

Write down the members of

1. G',
2. E',
3. $(G \cup E)'$.

How do the answers to 1.–3. change if $U = \{x | x$ is a country$\}$?

Exercise 9

We can use sets to denote sets of individuals, events, etc.

- "wears trousers" ...$\{x|x$ wears trousers$\}$,
- "cat" ...$\{x|x$ is a cat$\}$,

We can use this to represent meaning of sentences like

- "Cat meows" ...$\{x|x$ is a cat$\} \cap \{x|x$ meows$\}$.

Use this notation to express the meaning of the following sentences:

1. Two cats are meowing.
2. No dog barks.
3. Every dog barks.
4. All walls are white.

Set Equalities

Set Equalities

Let X, Y, Z be sets.

1. $X \cup X = X$, $X \cap X = X$
2. $X \cup Y = Y \cup X$, $X \cap Y = Y \cap X$
3. $X \cup \emptyset = X$, $X \cap \emptyset = \emptyset$
4. $(X \cup Y) \cup Z = X \cup (Y \cup Z) = X \cup Y \cup Z$
5. $(X \cap Y) \cap Z = X \cap (Y \cap Z) = X \cap Y \cap Z$

Distributive Property, Complement

Distributive Property

Let A, B, C be sets.

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Complements

Let A, B be sets and U the universe *(of discourse)*.

3. $A \cup A' = U$
4. $(A')' = A$
5. $A \cap A' = \emptyset$
6. $A - B = A \cap B'$

DeMorgan's Laws

Let A and B be sets.

- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

Exercise

Exercise 10

For sets A, B, C, show that

1. $(A \cap B) \cap (A \cap C)' = A \cap (B - C)$,
2. $A \cap (B - A) = \emptyset$.